Cayley Graphs and Thompson’s Group
**Cayley Graphs**

Cayley graphs are geometric objects used to study finitely generated groups. Let $G$ be a group and let $S$ be a finite generating set for $G$. Then the associated Cayley graph $\Gamma$ have vertices associated to the elements of $G$ and (directed) edges corresponding to right multiplication by the elements of $S$. 

![Diagram of Cayley Graphs](image-url)
Dihedral Groups

Dihedral groups are generated by a reflection and a (minimal, non-trivial) rotation. The associated Cayley graph for $D_4$ is:

![Cayley graph of D4](image)

This highlights that Cayley graphs often illuminate the geometry underpinning a group.
Alternating Groups

The alternating groups are key examples of finite simple groups. One of this weekend’s speakers, Martin Kassabov, has created a family of expander graphs using Cayley graphs of Alternating groups. Below is the Cayley graph of $A_4$ using (123) and (12)(34) as generators. (The picture is stolen from Wolfram’s Mathworld.)
Infinite Groups

While the main questions about Cayley graphs of finite groups are combinatorial, the main questions about Cayley graphs of infinite groups are geometric.
Another Infinite Group

Here’s the Cayley graph of a reflection group:
Geometric Questions

Many questions are framed in terms of “balls” and “spheres” in the Cayley graph.
Geometric Questions

1) Growth?
2) “Convexity”?
Thompson’s Group

Thompson’s group $F$ consists of certain piecewise linear functions from the closed interval $[0, 1]$ to $[0, 1]$: 

![Graph of a piecewise linear function]

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Generators?

Every element of $F$ can be expressed as a product of the following two elements and their inverses:
Strange Facts ...

So $F \oplus F < F$!

Sean Cleary (and others): The Cayley graph of Thompson's group
1) has “pockets”
2) has no “convexity”