Indexing and Hashing

- Cost estimation
- Basic Concepts
- B^+ - Tree Index Files
- Hashing
- Comparison of Ordered Indexing and Hashing

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Estimating Costs

- For simplicity we estimate the cost of an operation by counting the number of blocks that are read or written to disk.
- We ignore the possibility of blocked access which could significantly lower the cost of I/O.
- We assume that each relation is stored in a separate file with *B* blocks and *R* records per block.

Basic Concepts

- Indexing is used to speed up access to desired data.
  - » E.g. author catalog in library
- A search key is an attribute or set of attributes used to look up records in a file. Unrelated to keys in the db schema.
- An index file consists of records called index entries.
- An index entry for key *k* may consist of
  - » An actual data record (with search key value k)
  - » A pair (k, rid) where rid is a pointer to the actual data record
  - » A pair (k, bid) where bid is a pointer to a bucket of record pointers
- Index files are typically much smaller than the original file if the actual data records are in a separate file.
- If the index contains the data records, there is a single file with a special organization.

Index Evaluation Metrics

- Access time for:
  - » Equality searches – records with a specified value in an attribute
  - » Range searches – records with an attribute value falling within a specified range.
- Insertion time
- Deletion time
- Space overhead

Types of Indices

- The records in a file may be unordered or ordered sequentially by some search key.
- A file whose records are unordered is called a heap file.
- If an index contains the actual data records or the records are sorted by search key in a separate file, the index is called clustering (otherwise non-clustering).
- In an ordered index, index entries are sorted on the search key value. Other index structures include trees and hash tables.
- A primary index is an index on a set of fields that includes the primary key. Any other index is a secondary index.

B-Trees

- B-Trees are balanced search trees
  - » They are designed to be stored on external storage, e.g., magnetic disks
  - » They are designed to minimize disk accesses
  - » They are widely used in database systems
- B^+-Tree is a popular variant of the original B-Tree
  - » The keys are stored in the leaves

B$^+$-Tree Index Files

A B$^+$-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children where n is the maximum number of pointers per node.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and n – 1 values
- Special cases: if the root is not a leaf, it has at least 2 children. If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and n values.

B$^+$-Tree Node Structure

- A typical node has tens to hundreds of elements

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

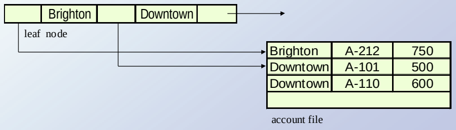
  - » K_i are the search-key values
  - » P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

Leaf Nodes in B$^+$-Trees

Properties of a leaf node:

- For i = 1,2,…, n-1, pointer P_i either points to a file record with search-key value K_i, or to a bucket of pointers to file records, each record having search-key value K_i. If L_i, L_j are leaf nodes and i < j, L_i's search-key values are less than L_j's search-key values
- P_n points to next leaf node in search-key order



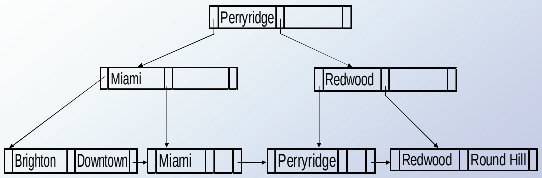
Brighton	A-212	750
Downtown	A-101	500
Downtown	A-110	600

Non-Leaf Nodes in B$^+$-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with m pointers:
  - » All the search-keys in the subtree to which P_i points are less than K_i
  - » All the search-keys in the subtree to which P_i points are greater than or equal to K_{i-1}

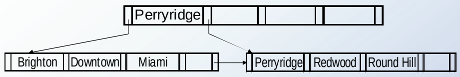
P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

Examples of a B$^+$-tree



B$^+$-tree for account file (n=3)

Example of a B$^+$-tree



B$^+$-tree for account file (n=5)

- Leaf nodes must have between 2 and 4 values ($\lceil (n-1)/2 \rceil$ and n – 1, with n=5).
- Non-leaf nodes other than root must have between 3 and 5 children ($\lceil n/2 \rceil$ and n with n = 5).
- Root must have at least 2 children

Observations about B^+ -trees

- Since the inter-node connections are done by pointers, there is no assumption that in the B^+-tree , the "logically" close blocks are "physically" close.
- The B^+-tree contains a relatively small number of levels (logarithmic in the size of the main file), thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B^+-Trees

- Find all records with a search-key value of k.
  - » Start with the root node
  - » Examine the node for the smallest search-key value > k.
  - » If such a value exists, assume it is K_i. Then follow P_i to the child node
  - » Otherwise $k \geq K_{m-1}$, where there are m pointers in the node, Then follow P_m to the child node.
  - » If the node reached by following the pointer above is not a leaf node, repeat the above procedure on the node, and follow the corresponding pointer.
  - » Eventually reach a leaf node. If key $K_i = k$, follow pointer P_i to the desired record or bucket. Else no record with search-key value k exists.

Queries on B^+-Trees (Cont.)

- In processing a query, a path is traversed in the tree from the root to some leaf node.
- If there are K search-key values in the file, the path is no longer than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes, and n is typically around 200 (20 bytes per index entry).
- With 1 million search key values and n = 200, at most $\log_{100}(1{,}000{,}000) = 3$ nodes are accessed in a lookup.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - » above difference is significant since every node access may need a disk I/O, costing around 20 millisecond!

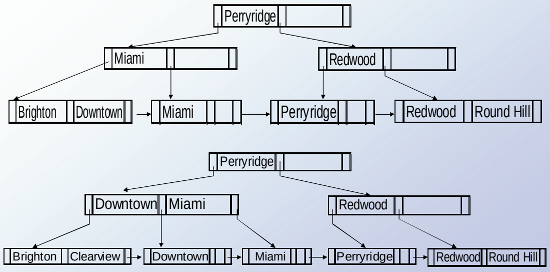
Updates on B^+-Trees : Insertion

- Find the leaf node in which the search-key value would appear
- If the search-key value is already there in the leaf node, record is added to file and if necessary pointer is inserted into bucket.
- If the search-key value is not there, then add the record to the main file and create bucket if necessary. Then:
  - » if there is room in the leaf node, insert (search-key value, record/bucket pointer) pair into leaf node at appropriate position.
  - » if there is no room in the leaf node, split it and insert (search-key value, record/bucket pointer) pair as discussed in the next slide.

Updates on B^+-Trees: Insertion (Cont.)

- Splitting a node:
  - » take the n(search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - » Let the new node be p, and let k be the least key value in p. Insert (k, p) in the parent of the node being split. If the parent is full, split it and propagate the split further up.
- The splitting of nodes proceeds upwards till a node that is not full is found. In the worst case the root node may be split increasing the height of the tree by 1.

Updates on B^+-Trees : Insertion(Cont.)



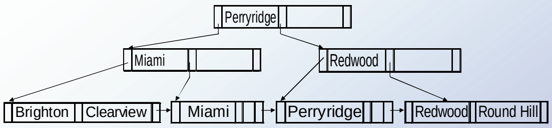
Updates on B$^+$-Trees : Deletion

- Find the record to be deleted, and remove it from the main file and from the bucket (if present)
- Remove (search-key value, pointer) from the leaf node if there is no bucket or if the bucket has become empty
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then
  - » Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - » Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B$^+$-Trees : Deletion

- Otherwise, if the node has too few entries due to the removal, and the entries in the node and a sibling don't fit into a single node, then
  - » Redistribute the pointers between the node and a sibling such that both have at least the minimum number of entries
  - » Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found. If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

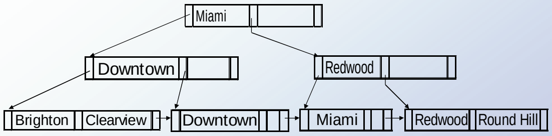
Examples of B$^+$-Tree Deletion



Result after deleting "Downtown" from account

- The removal of the leaf node containing "Downtown" did not result in its parent having too little pointers. So the cascaded deletions stopped with the deleted leaf node's parent.

Examples of B$^+$-Tree Deletion (Cont.)



Deletion of "Perryridge" instead of "Downtown"

- The deleted "Perryridge" node's parent became too small, but its sibling did not have space to accept one more pointer, so redistribution is performed. Observe that the root node's search-key value changes as a result.

B$^+$-Tree File Organization

- Index file degradation problem is solved by using B$^+$-Tree indices. Data file degradation problem is solved by using B$^+$-Tree File Organization.
- The leaf nodes in a B+-tree file organization store records, instead of pointers.
- Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Leaf nodes are still required to be half full.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B$^+$-tree index.
- Good space utilization is important since records use more space than pointers. To improve space utilization, involve more sibling nodes in redistribution during splits and merges.

Static Hashing

- A **bucket** is a unit of storage containing one or more records (a bucket is typically a disk block). In a **hash file organization** we obtain the bucket of a record directly from its search-key value using a **hash function**.
- Hash function h is a function from the set of all search-key values K to the set of all bucket addresses B.
- Hash function is used to locate records for access, insertion, and deletion.
- Records with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate a record.

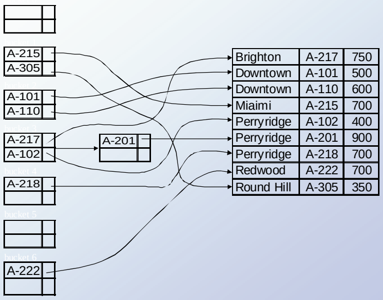
Hash Functions

- Worst hash function maps all search-key values to the same bucket; this makes access time proportional to the number of search-key values in the file.
- An ideal hash function is *uniform*, i.e. each bucket is assigned the same number of search-key values from the set of all possible values.
- Ideal hash function is *random*, so each bucket will have the same number of records assigned to it irrespective of the actual distribution of search-key values in the file.
- Typical hash functions perform computation on the internal binary representation of the search-key. For example, for a string search-key, the binary representations of all the characters in the string could be added and the sum modulo number of buckets could be returned.

Hash Indices

- Hashing can be used not only for file organization, but also for index-structure creation. A **hash index** organizes the search keys, with their associated record pointers, into a hash file structure.
- Hash indices are always secondary indices — if the file itself is organized using hashing, a separate primary hash index on it using the same search-key is unnecessary. However, the term hash index is used to refer to both secondary index structures and hash organized files.

Example of Hash Index



Deficiencies of Static Hashing

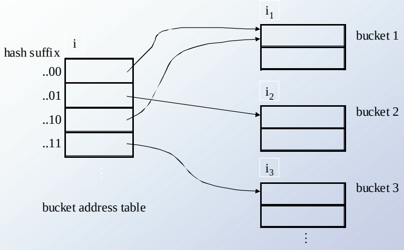
- In static hashing, function h maps search-key values to a fixed set B of bucket addresses.
  - » Databases grow with time. If initial number of buckets is too small, performance will degrade due to too much overflows.
  - » If file size at some point in the future is anticipated and number of buckets allocated accordingly, significant amount of space will be wasted initially.
  - » If database shrinks, again space will be wasted.
  - » One option is periodic, re-organization of the file with a new hash function, but it is very expensive.
- These problems can be avoided by using techniques that allow the number of buckets to be modified dynamically.

Dynamic Hashing

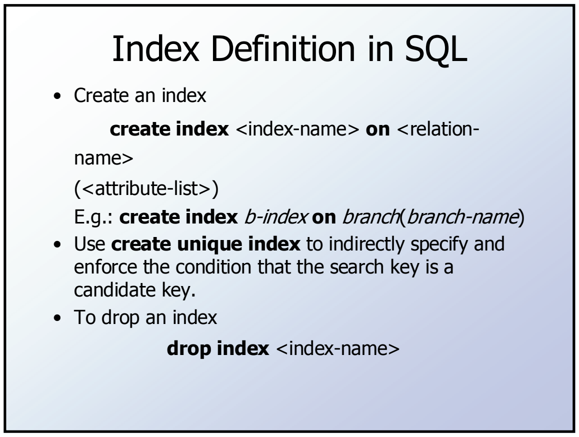
- Good for database that grows and shrinks in size
- Allows the hash function to be modified dynamically
- **Extendable hashing** – one form of dynamic hashing
  - » Hash function generates values over a large range — typically

    b-bit integers, with *b* = 32.
  - » At any time use only the last *i* bits of the hash function to index into a table of bucket addresses, where: $0 \leq i \leq 32$
  - » Initially i = 0
  - » Value of i grows and shrinks as the size of the database grows and shrinks.
  - » Actual number of buckets is $< 2^i$, and this also changes dynamically due to merging and splitting of buckets.

General Extendable Hash Structure



In this structure, $i_2 = i_3 = i$, whereas $i_1 = i - 1$



Index Definition in SQL

- Create an index

  create index <index-name> **on** <relation-name>

  (<attribute-list>)

  E.g.: **create index** *b-index* **on** *branch*(*branch-name*)
- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key.
- To drop an index

  drop index <index-name>